3.1.4 Journal: Proving the Pythagorean Theorem

Mathematics II Sem 2

Journal

Name: Redacted
Date:

Scenario: Will the TV Fit?

Instructions:

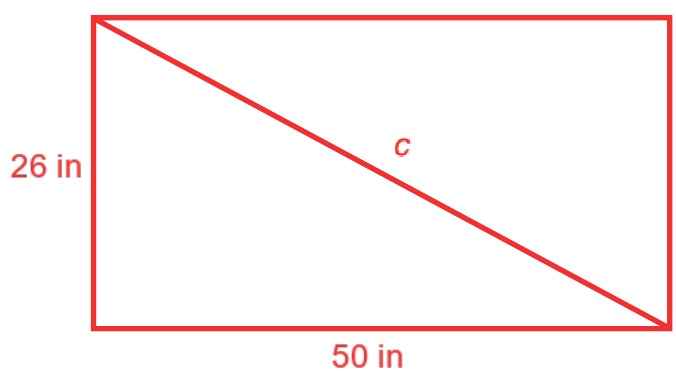
- View the video found on page 1 of this Journal activity.
- Using the information provided in the video, answer the questions below.
- Show your work for all calculations

1.

The Students' Conjectures (3 points: 1 point each)	
What conjecture is being made?	It is possible to fit the TV in diagonally.
What key details are given?	The trunk is 26 inches tall and 50 inches wide. The TV is 96 inches long and 54 inches wide.
How will you determine if the conjecture is true?	Use similar triangles to determine the longest diagonal in the trunk.

2. Determine if the TV will fit in the car.

a. Draw the rectangle that represents the height and width of the trunk including the diagonal. Label the dimensions on your sketch. (**1 point**)



b. Use the Pythagorean theorem to calculate the length of the diagonal. (**2 points**)

$a^2 + b^2 = c^2$

$50^2 + 26^2 = c^2 \Rightarrow 2500 + 676 = c^2 \Rightarrow c^2 = 3176 \Rightarrow c = \sqrt{3176} \Rightarrow c \approx 56.36$

$c = \sqrt{4 * 794} \Rightarrow c = 2\sqrt{794}$

$a = 50$ inches

$b = 26$ inches

Radical form (exact): $\mathbf{c = \sqrt{3176}}$

Simplified radical form (exact): $\mathbf{c = 2\sqrt{794}}$

Numerical form (approximate): $\mathbf{c \approx 56.36}$

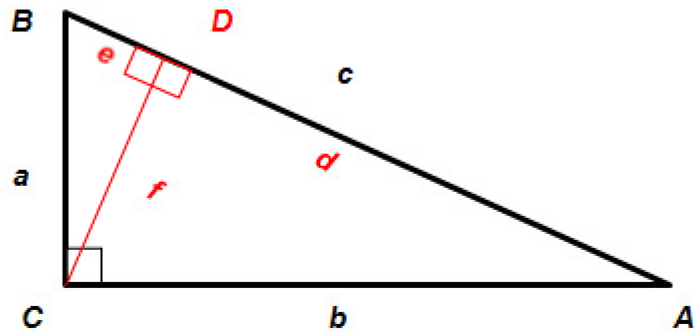
c. Assuming the trunk is at least 96 inches deep, can the TV fit into the trunk of the car? Explain your answer. (**1 point**)

Yes, most certainly. By definition of a right triangle, any one leg is shorter than the hypoteneuse, but the sum of the legs is longer. If the depth of the trunk was 96 inches, and the diagonal was 56.36 inches, the TV would fit.

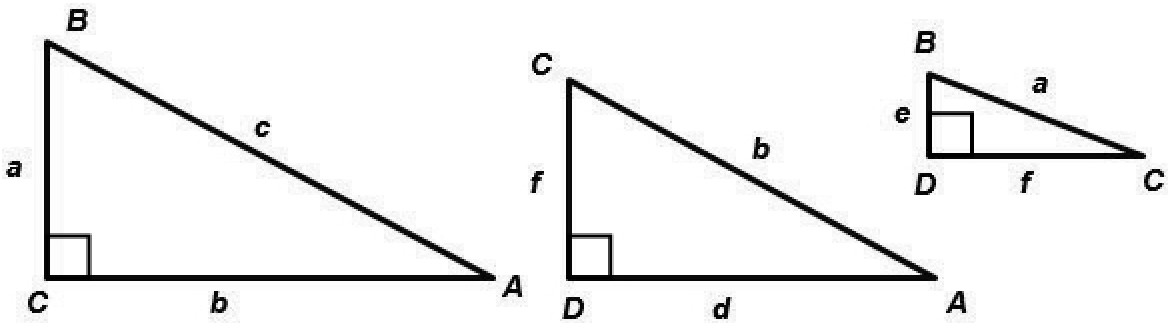
3. The Similar Triangle Method (**9 points total**)

To prove that the Pythagorean theorem works, leave out the dimensions and represent the trunk of the car with a right triangle *ABC*.

Drawing an altitude from vertex *C* to side *c* creates two new triangles, *BCD* and *ACD* (see diagram).



a. Here are the three triangles shown separately These triangles are similar. How do you know? HINT: Compare the angle measures. (**1 point**)



Click here for long description

ΔBCD is similar to ΔBAC by AA similarity;
- ∠BDC = ∠BCA (right angels)
- ∠DBC = ∠DBC (∠CBA) (reflexive property / same angle)

ΔCAD is similar to ΔBAC by AA similarity;
- ∠CDA = ∠BCA (right angles)
- ∠CAD = ∠CAD (∠BAC) (reflexive property / same angle)

ΔBCD is similar to ΔCAD by the Transitive Property
- ΔBCD ~ ΔBAC and ΔCAD ~ ΔBAC, thus, by substitution ΔBCD ~ ΔCAD

b. Complete the proportion to compare the first two triangles. (**1 point**)

$$\frac{b}{c} = \frac{\boxed{d}}{\boxed{b}}$$

c. Cross-multiply the ratios in part b to get a simplified equation. (**1 point**)

$b^2 = cd$

d. Complete the proportion to compare the first and third triangles. (**1 point**)

$$\frac{c}{a} = \frac{\boxed{a}}{\boxed{e}}$$

e. Cross multiply the ratios in part d to get a simplified equation. (**1 point**)

$a^2 = ce$

f. Complete the steps to add the equations from parts c and e. This will make one side of the Pythagorean theorem. (**1 point**)

$$\begin{aligned} \textbf{Part c}: b^2 &= \underline{cd\qquad\quad} \\ \textbf{Part e}: a^2 &= \underline{ce\qquad\quad} \\ a^2 + b^2 &= \underline{cd + ce\quad} \end{aligned}$$

g. Factor out a common factor from part f. (**1 point**)

$$a^2 + b^2 = \underline{c\quad}(\underline{d\quad} + \underline{e\quad})$$

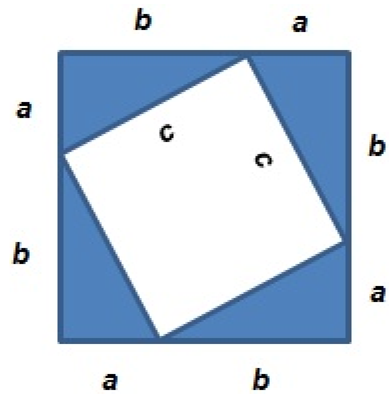
h. Finally, replace the expression inside the parentheses with one variable and then simplify the equation to a familiar form. HINT: Look at the large triangle at the top of this problem. (**2 points**)

$$\begin{aligned} a^2 + b^2 &= \underline{c\quad}(\underline{c\quad}) \\ a^2 + b^2 &= \underline{c^2\qquad} \end{aligned}$$

Further Exploration

4. A Geometric Proof (**4 points total**)

Here is another interesting proof. The smaller white square is inside a larger square. Each blue area is a right triangle. To find the area of the white square, you can find the area of the big square and subtract the areas of the right triangles.



$$\text{Area of big square} = (a+b)^2$$
$$\text{Area of 4 triangles} = 4\left(\tfrac{1}{2}ab\right)$$
$$\text{Area of white square} = \text{big square - 4 shaded triangles}$$

a. Use these areas to write an equation for the area of the white square. Simplify the equation if possible. (**2 points**)

$(a + b)^2 - 4(½ab) = c^2 \Rightarrow (a^2 + 2ab + b^2) - 2ab = c^2 \Rightarrow a^2 + b^2 = c^2$

b. Compare the similar triangle proof from question 3 with the inscribed square proof. How are they different? Which method was easier for you to understand? (**1 point**)

The triangle similarity approach offered a more logical and algebraic approach while the inscribed square proof gave a direct example of how a right triangle relates to the square. Overall, I found both approachs easy and simple to understand.

c. Are there any other considerations that should be taken into account when trying to fit a giant TV into your car? (**1 point**)

The third dimension, thickness (height), is of paramount importance to take into consideration. Although most TVs these days are thin flat-screens, it should be noted that they still might not fit. Furthermore, a 54 by 96 inch TV is absolutely MASSIVE. That's quite a bit larger than a queen-size bed.
